

MERGING RATES OF COMPACT BINARIES IN THE UNIVERSE: GRAVITATIONAL WAVES AND GAMMA-RAY BURSTS

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Merging rates of compact binaries (double neutron stars or black holes) are calculated based on the modern concept of binary stellar evolution. It is found that the initial laser interferometers with an rms-sensitivity of 10^{-21} at the frequency 100 Hz can detect 10–700 black holes and only ~ 1 neutron star coalescences in a 1-year integration time. Implications of the evolutionary effects to the cosmological origin of GRB are also discussed.

1 Introduction

In a few years several initial ground-based laser interferometers aimed at searching for gravitational waves (GW) will start working (LIGO¹, VIRGO², GEO-600³, TAMA-300⁴), so at present the question about what kind of events and how frequently will the interferometer register is very important.

Undoubtedly, the most reliable GW sources are merging compact binary stars – double neutron stars (NS) and black holes (BH) of different stellar masses. In the same time, merging compact binaries may underly the origin of cosmic gamma-ray bursts (GRB) (Blinnikov *et al.*⁵, Paczyński⁶, Meszaros and Rees⁷).

In Section 2 we briefly describe observational and theoretical data about double degenerate compact binaries and estimates of their galactic merging rates. In Section 3 we discuss evolutionary scenario parameters that mostly affect binary NS and BH merging rates. The results of calculations of the merging rates in our Galaxy are presented in Section 4. Then we discuss the transition from the merging rates in an individual galaxy to the detection rate by a GW detector with characteristics similar to those of the initial LIGO/VIRGO interferometer (this is the most important result of the paper about GW events). Two last parts of the paper deal

The contribution presented by M.E. Prokhorov

with GRB (Section 5) and unidentified soft X-ray flashes (Section 6) recently reported in the Einstein IPC archive studies⁸.

2 The Present Situation in the Galaxy

2.1 Observational Data

At present, the following facts are known about double degenerate compact binaries in our Galaxy:

Table 1: Binary PSR with NS secondaries (Nice *et al.*⁹)

PSR	$P_b(d)$	e	M_1	M_2	τ , yr
J1518+4904	8.634	0.249	∞
B1913+16 ^a	0.323	0.617	1.44	1.39	$3 \cdot 10^8$
B1534+12 ^a	0.420	0.274	1.34	1.34	$1.5 \cdot 10^9$
B2127+11c ^a	0.335	0.681	1.35	1.36	$3 \cdot 10^8$
B2303+46	12.340	0.658	∞
B1820-11 ^b	357.762	0.795	∞

^a Coalescing pulsars

^b The secondary component may be not a NS

Table 2: BH Candidates (Cherepashchuk¹⁰).

System	Sp. class	P_{orb} , d	$f_v(m), M_\odot$	m_x, M_\odot	m_v, M_\odot
Cyg X-1	O9,7 Iab	5.6	0.23	7–18	20–30
LMC X-3	B(3–6)II–III	1.7	2.3	7–11	3–6
LMC X-1	O(7–9)III	4.2	0.14	4–10	18–25
A0620-00	K(5–7)V	0.3	3.1	5–17	~0.7
GS2023+338	K0IV	6.5	6.3	10–15	0.5–1.0
GSR1121-68	K(3–5)V	0.4	3.01	9–16	0.7–0.8
GS2000+25	K(3–7)V	0.3	5.0	5.3–8.2	~0.7
GRO J0422+32	M(0–4)V	0.2	0.9	2.5–5.0	~0.4
GRO J1655-40	F5IV	2.6	3.2	4–6	~2.3
XN Oph 1977	K3	0.7	4.0	5–7	~0.8
Mean BH mass	~8.5				

1. A few binary radiopulsars are known to have the secondary NS component (Table 1).
2. Three of these binary pulsars must coalesce due to the orbital angular momentum removal by GW on a time scale shorter than the age of the Universe (the Hubble time $t_H \simeq 15 \cdot 10^9$ yr).
3. No binary pulsars with BH is known as yet (although from evolutionary considerations one may expect one such an object to be formed in the Galaxy per about 1000 single pulsars, Lipunov *et al.*¹¹)
4. No binary BH has been found so far.

5. In contrast, 10 BH candidates are already known in X-ray binary systems with normal companions¹⁰. Their parameters are listed in Table 2. Note that the mean BH mass in these systems is $\langle M_{bh} \rangle \simeq 8.5 M_{\odot}$, i.e. BH formed in stellar evolution are notably more massive than NS (with the typical mass $1.4 M_{\odot}$).

2.2 Binary NS Merging Rate Estimates

At present, it is possible to estimate binary NS merging rate in two ways: using the binary radiopulsar statistics observed and making various computations of binary stellar evolution.

“Observational” estimates

Phinney ¹²	$1/10^6$ yr
Narayan <i>et al.</i> ¹³	$1/10^6$ yr
Curran and Lorimer ¹⁴	$3/10^6$ yr
van den Heuvel and Lorimer ¹⁵	$8/10^6$ yr
“Bailes limit” ¹⁶	$< 1/10^5$ yr

“Theoretical” estimates

Clark <i>et al.</i> ¹⁷	$1/10^4$ – $1/10^6$ yr
Lipunov <i>et al.</i> ¹⁸	$1/10^4$ yr
Hils <i>et al.</i> ¹⁹	$1/10^4$ yr
Tutukov and Yungelson ²⁰	$3/10^4$ – $1/10^4$ yr
Lipunov <i>et al.</i> ²¹	$< 3/10^4$ yr
Portegies Zwart and Spreeuw ²²	$3/10^5$ yr
Lipunov <i>et al.</i> ²³	$3/10^4$ – $3/10^5$ yr

We emphasize that although theoretical merging rates are systematically higher than observational ones, both estimates do not contradict each other. The main argument is that the first (observational) estimates of binary NS merging rate are based on the statistics of binary systems, in which only one of the components shines as radiopulsar, which is not at all the *necessary* condition for merging to occur. Thus the observational estimates are in fact only lower limits. There are also some selection effects that can change estimates from both groups.

3 Parameters of binary compact star evolution

To calculate binary evolution, we have used the population synthesis method (the Scenario Machine code), which is in fact a version of the Monte-Carlo calculations. Here we shall not enter into detail of the evolutionary scenario used. Much more detailed description of the method can be found in our review²⁴.

An example of the evolutionary track leading to BH+BH binary system formation is shown in Fig. 1. A short glance to this track is sufficient to understand that there are a lot of evolutionary scenario parameters, which affect different stages of the binary evolution. Fortunately, a very limited number of parameters has effect on the compact binary merging rate.

The most important (and practically unique) parameter changing the galactic binary NS merging rate is the distribution of an additional (kick) velocity imparted to NS at birth. The kick velocity distribution widely accepted at present is derived from the analysis of spatial velocities of single radiopulsars²⁵.

We have approximated this 3-dimensional distribution as

$$f_{LL}(x)dx \propto x^{0.19}(1+x^{6.72})^{-1/2}dx$$

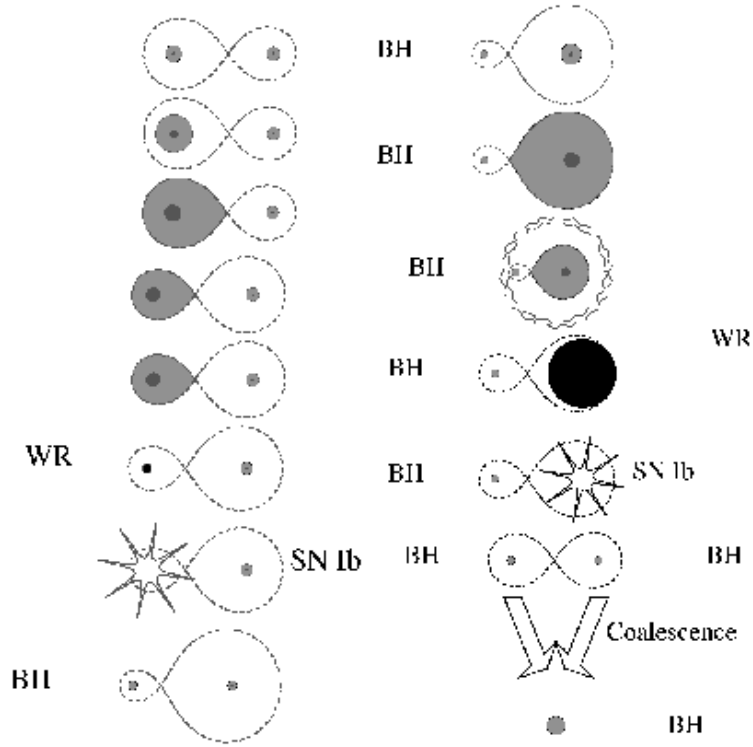


Figure 1: Massive binary evolutionary track.

where $x = w/w_0$ and the characteristic velocity w_0 is a parameter in our calculations. The observed pulsar transverse velocity distribution corresponds to $w_0 = 400$ km/s.

In contrast, for BH two additional parameters appear. First of them is a threshold main sequence stellar mass M_{cr} for the star to collapse into a BH after its nuclear evolution has ended. This parameter is still poorly determined and varies in a wide range: e.g., according to van den Heuvel and Habets²⁶ $M_{cr} = 40\text{--}80M_\odot$; Tsujimoto *et al.*²⁷ give $40\text{--}60M_\odot$; Portegies Zwart *et al.*²⁸ derive $>20M_\odot$.

The second parameter is the fraction of the presupernova mass, k_{BH} , collapsing into BH. This parameter is fully unknown, so we varied it from 0.1 to 1 in our calculations.

The third parameter, as for NS, is the kick velocity. Clearly, in the general case the more massive BH will acquire smaller velocities than NS. In our calculation we used the following *ad hoc* relationship

$$\frac{w_{BH}}{w_{NS}} = \frac{M_{preSN} - M_{BH}}{M_{preSN} - M_{OV}},$$

where $M_{OV} = 2.5M_\odot$ is the maximal NS mass (Oppenheimer-Volkoff limit). When BH mass is close to M_{OV} , velocities of BH and NS are assumed to be almost the same, whereas at $k_{BH} = 1$ BH kick velocity is assumed to vanish. (Of course, other dependences w_{BH}/w_{NS} are possible, but their specific shape weakly affects the results).

4 Detection rate of binary compact star merging

Under the assumptions made above, we can calculate the binary merging rates in the Galaxy R . The results are presented in Fig. 2 and 3

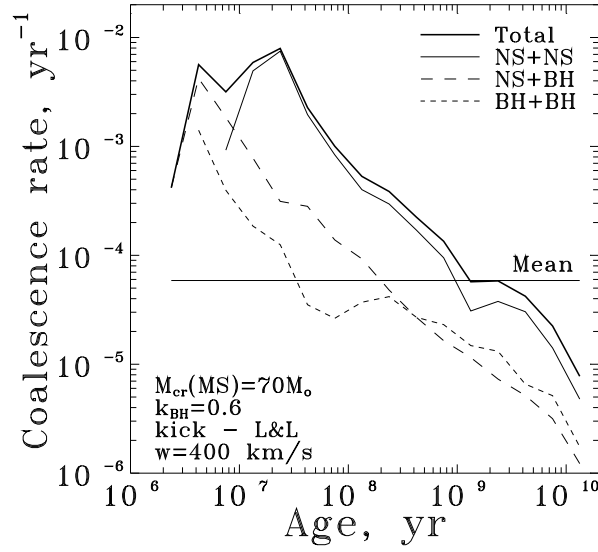


Figure 2: The dependence of different compact binary merging rates in a model “elliptical” galaxy (where all the stars were formed simultaneously at the moment $t = 0$) with mass $M = 10^{11} M_{\odot}$. The horizontal line shows the mean merging rate of binary NS in a spiral galaxy of the same mass with a stationary star formation.

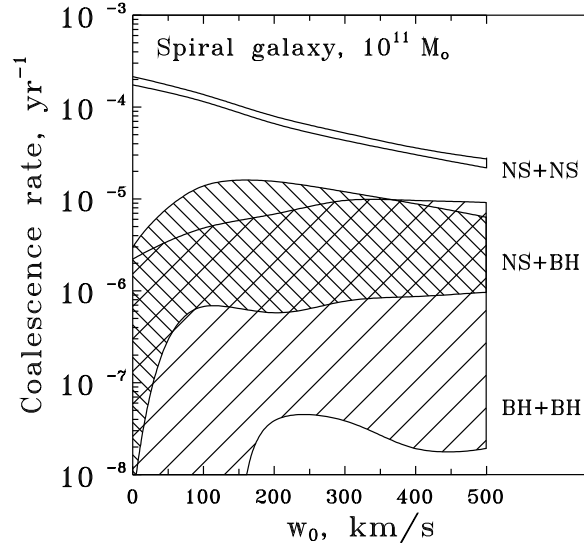


Figure 3: The dependence of different compact binary merging rates in a spiral galaxy with $10^{11} M_{\odot}$ on the characteristic kick velocity w_0 .

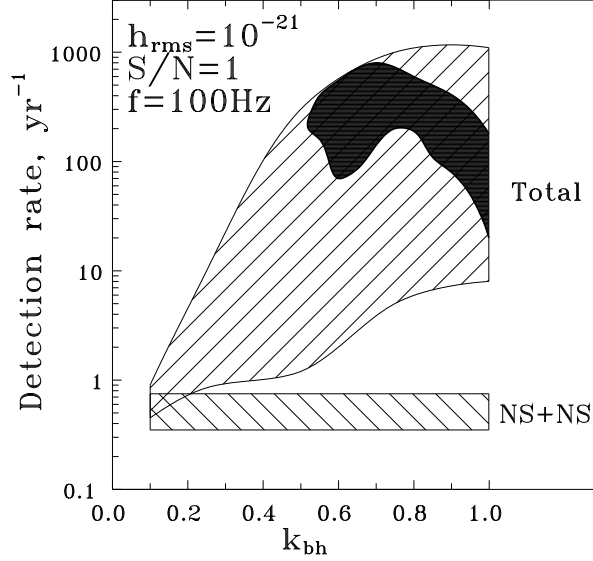


Figure 4: The total merging rate of NS+NS, NS+BH, and BH+BH binaries as would be detected by a laser interferometer with $h_{rms} = 10^{-21}$ for Lyne–Lorimer kick velocity distribution with $w_0 = 200\text{--}400$ km/s and BH progenitor’s masses $M_* = 15\text{--}50M_\odot$, for different scenarios of binary star evolution as a function of k_{bh} . NS+NS mergings are shown separately. In all cases BH+BH mergings contribute more than 80% to the total rate. The filled “Loch–Ness–monster–head”-like region corresponds to BH formation parameters $M_* > 18M_\odot$ and $k_{bh} = 0.5$.

After having found the merging rates R in a typical galaxy, we need to go over the event rate D at the detector. Applying the optimal filtering technique²⁹, the signal-to-noise ratio S/N at the spiral-in stage is

$$\frac{S}{N} \propto \frac{M_h^{5/6}}{d}.$$

Here $M_{ch} = (M_1 M_2)^{3/5} (M_1 + M_2)^{2/5}$ is “chorp”-mass of the binary system. This means that for a given S/N our detector can register more massive BH from larger distances than NS. The volume within which BH or NS is to be detected should be proportional $M_{ch}^{15/6}$. Then the ratio of detection rates of BH and NS can be written as

$$\frac{D_{BH}}{D_{NS}} = \frac{R_{BH}}{R_{NS}} \left(\frac{M_{BH}}{M_{NS}} \right)^{15/6}.$$

Let us make a simple estimate. Take a GW detector with $h_{rms} = 10^{-21}$ and $S/N = 1$ (as for the initial LIGO or VIRGO interferometer). Let the mass of NS and BH be $1.4M_\odot$ (the typical value well justified experimentally) and $8.5M_\odot$ (the mean mass of the BH candidates, see Table 2). Let us also assume that any star with the initial mass $M(NS) > 10M_\odot$ yields NS (the typical value confirmed theoretically), and the threshold mass for BH formation is the maximal from the estimates given above, $M(BH) > M_{cr} = 80M_\odot$. Hence using Salpeter mass function we find

$$\frac{R_{BH}}{R_{NS}} \simeq \frac{N(M > 80M_\odot)}{N(M > 10M_\odot)} = \left(\frac{80M_\odot}{10M_\odot} \right)^{-1.35} \simeq 0.06$$

and then

$$\frac{D_{BH}}{D_{NS}} = \left(\frac{80M_\odot}{10M_\odot} \right)^{-1.35} \left(\frac{8.5M_\odot}{1.40M_\odot} \right)^{15/6} \simeq 0.06 \times 90 \simeq 5$$

This estimate is, of course, very rough, but the precise calculation gives essentially the same result. Fig. 4 shows the calculated absolute registration rate of mergings D by the detector

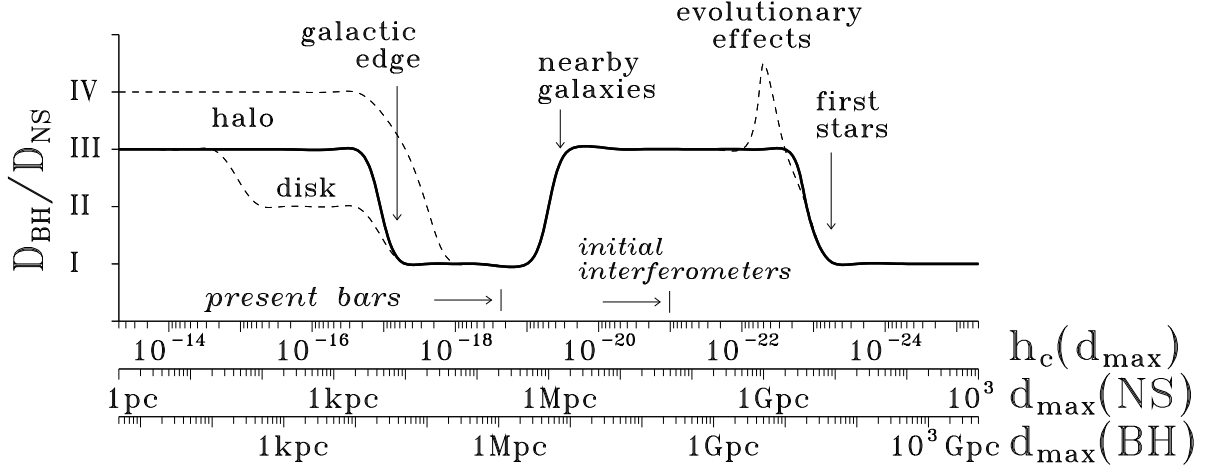


Figure 5: A schematic dependence of the detection rate ratio on the detector sensitivity (the upper scale) and the limiting distance from which binary NS ($1.4 + 1.4M_{\odot}$, the middle scale) and binary BH ($8.5 + 8.5M_{\odot}$, the bottom scale) mergings can be detected. See the text for more detail.

with $h_{rms} = 10^{-21}$ and $S/N = 1$, as a function of k_{BH} . The calculations have been done for Lyne–Lorimer kick velocity distribution with the parameter $w_0 = 400$ /. The vertical dispersion is due to the dependence of D on M_{cr} . It is seen that binary NS mergings occur with a rate of about 1 event per year. In the same time, the total detection rate (except for $k_{BH} \approx 0.1$) can exceed that of binary NS mergings by 2–3 orders. The filled region shows the region of the “most probable parameters”, for which the calculations meet both the upper limit to the number of BH with radiopulsars (less than 1 per 700 single pulsars) and the number of BH candidates of Cyg X-1 type (from 1 to 10 in the Galaxy). In this region, from 10 to 100 mergings per year must be detected, with the majority of them being binary BH mergings.

4.1 Discussion

Nevertheless, one may wonder whether binary BH (which are less numerous than binary NS) should always be detected more frequently than binary NS. It seems worth looking more closely on the stellar mass distribution around a GW detector on Earth (Fig. 5). In this Figure, the detection rate ratio D_{BH}/D_{NS} is plotted schematically against the detector sensitivity level (the upper scale), which can be expressed through the maximum distance from which binary NS/BH mergings with $M_{NS} = 1.4M_{\odot}$ and $M_{BH} = 8.5M_{\odot}$ can be detected (two bottom scales). Four segments may be distinguished in this plot (from left to right): first, when we register objects inside some part of the Galaxy, second, when all objects within the Galaxy are detected but no extragalactic objects can be detected, third, mainly extragalactic events are detected from distances more close than those at which the initial star formation occurs, and forth, where we detect all events in the Universe. In different segments different detection ratios will be obtained. In the first segment, the detection ratio depends on the galactic structure: if NS and BH populate the same spherical halo, this ratio (roman III on the vertical axis; the solid line) is $R_{BH}/R_{NS}(M_{BH}/M_{NS})^{15/6}$; if NS and BH populate the galactic disk, this ratio becomes (roman II, the bottom dashed line) $R_{BH}/R_{NS}(M_{BH}/M_{NS})^{10/6}$; if NS fill more extended halo than BH, i.e. the halo radius $r_{NS} > r_{BH}$ (roman IV, the upper dashed line), then the detection ratio is $R_{BH}/R_{NS}(M_{BH}/M_{NS})^{15/6}(r_{BS}/r_{NS})^3$. In the second and fourth segments the detection ratio will be minimal and simply equal to (roman I) R_{BH}/R_{NS} . In the third segment, the detection ratio of type III is realized. At the end of this segment evolutionary effects can affect the

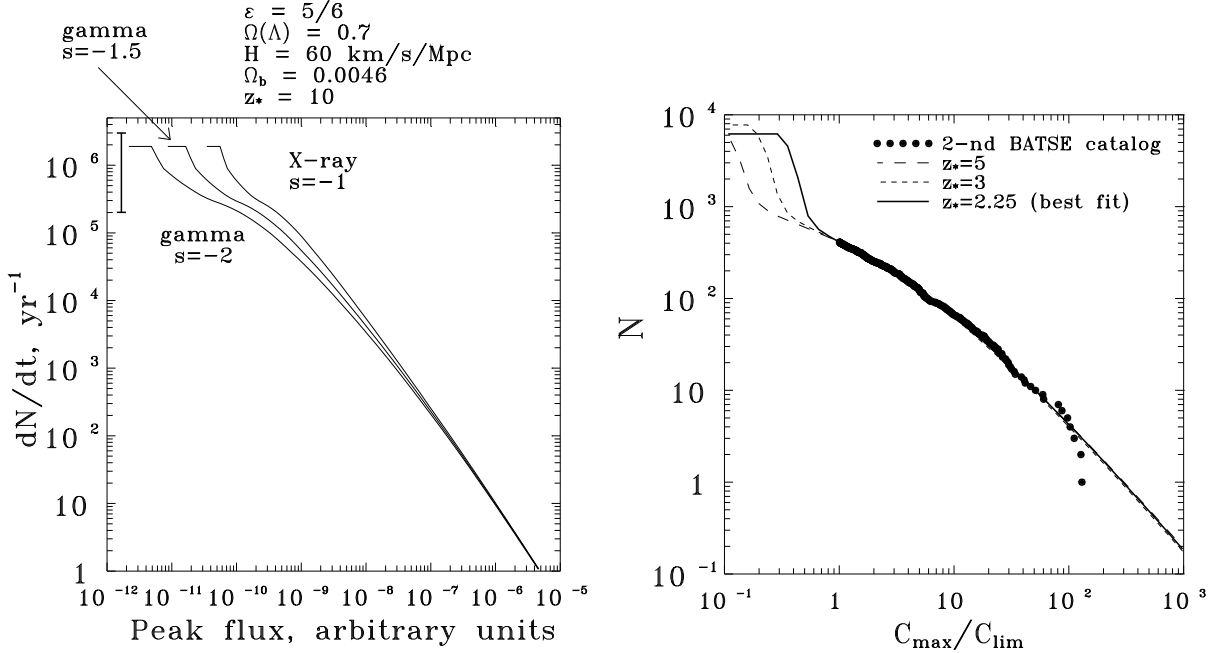


Figure 6: (Left panel) $\log N$ - $\log S$ curves calculated for different spectral power-law indices attributable to gamma- and X-ray emission in a GRB. Values of the cosmological model parameters are shown in the Figure. The bar indicates the accessible range of the total GRB rates in the entire Universe varying the parameters as discussed in the text.

Figure 7: (Right panel) The 2-nd BATSE catalog (solid points) is fitted by the cosmological GRB model (from Lipunov *et al.*³⁰). Note that the total GRB rate in the Universe is $\sim 10^4$ per year, ~ 3 orders of magnitude smaller than the total binary NS merging rate.

detection ratio (the dashed line).

Note that the present sensitivity of bar detectors ($h_{rms} \sim 10^{-19}$) falls within the second segment (an “unhappy” situation because no mass-ratio enhancement for BH detection occurs), whereas the initial laser interferometers ($h_{rms} \sim 10^{-21}$) are “luckily” in the third segment with the enhanced type III detection ratio. With the advanced LIGO sensitivity, it is possible to detect evolutionary effects and even reach the very edge of star formation.

5 Gamma-Ray Bursts

Using the obtained above dependence of compact binary merging rates in the elliptical galaxy on time (Fig. 2) and assuming the cosmological origin of GRB as products of binary NS/BH coalescences, we can compute the theoretical $\log N$ - $\log S$ curve. To do this, we need to specify the cosmological parameters, the moment of the star formation beginning, and the spectral power-law index of a typical GRB (see Lipunov *et al.*³⁰ for more detail). Taking the density of baryons in stars $\Omega_* = 0.0046$ (in terms of critical density to close the Universe)³¹ and varying other parameters within limits permitted by the present theory and observations (Λ -term: $0 \leq \Omega_\Lambda < 0.7$; the fraction of elliptical galaxies: $0.15 < \varepsilon < 0.9$; the star formation starting redshift: $2.5 < z_* < 10$), the total compact binary merging rate in the Universe (a constant the $\log N$ - $\log S$ curve goes at small fluxes) is found to vary not too much from a few $\cdot 10^5 \text{ yr}^{-1}$ to a few $\cdot 10^6 \text{ yr}^{-1}$ (see Fig. 6).

This curve is consistent with observational data obtained by BATSE (Fig. 7 from³⁰). However, the total GRB rate in the Universe will be of order $\sim 10^4 \text{ yr}^{-1}$. The difference in about 3

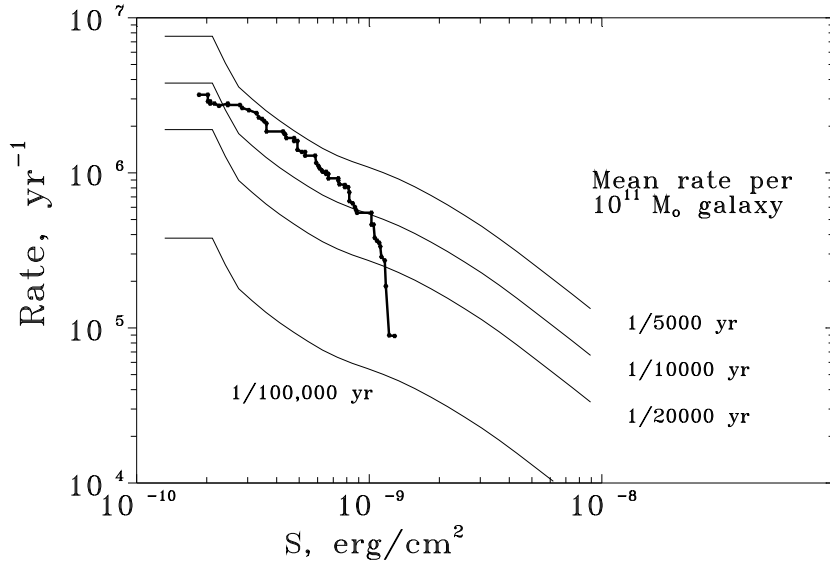


Figure 8: The observed $\log N$ - $\log S$ distribution for the Einstein IPC soft X-ray flashes (from Gotthelf *et al.*⁸; the broken line) and theoretical $\log N$ - $\log S$ curves (with the same parameters as in Fig. 6) for four different values of the mean binary NS merging rate in a spiral $10^{11} M_{\odot}$ galaxy.

orders can be explained either by assuming the only one merging of ~ 1000 to yield GRB, or by the gamma-ray emission collimation into a $\sim 7^{\circ}$ solid angle³⁰.

6 Einstein Soft X-ray Bursts

In 1996, after Einstein IPC archive data reprocessing, weak soft X-ray flashes were reported⁸, which are distributed isotropically on the sky and unidentified with known astronomical objects, like GRB. The interpolation of the Einstein IPC field of view on the total celestial sphere yields the rate of these flashes $\sim 2 \cdot 10^6 \text{ yr}^{-1}$. If the existence of these flashes is confirmed (for example, using independent ROSAT and ASCA data) and they are generated by binary NS coalescence, they will be a serious indication favouring the high, $\sim 1/10000 \text{ yr}^{-1}$, NS merging rate in the Galaxy (see Fig. 8).

7 Conclusion

To conclude, we expect the GW events due to compact binary coalescences to be registered already by the initial laser GW-interferometers of LIGO/VIRGO type at a rate substantially higher than has previously been thought. We also think that BH will thus be detected simultaneously with GW. Otherwise, we will have very stringent constraints on BH formation parameters, which is, of course, much less interesting.

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